Available online at www.sciencedirect.com
ScienceDirect
Applied Radiation and Isotopes 65 (2007) 454–457
Applied Radiation and Isotopes
www.elsevier.com/locate/apradiso

Technical note

Semi-empirical systematics of (n,^{3}He) reaction cross-section at 14.6 and 20 MeV

C.H.M. Broeders, A.Yu. Konobeyev*

Institut für Reaktorsicherheit, Forschungszentrum Karlsruhe GmbH, 76021, Karlsruhe, Germany

Received 22 May 2006; received in revised form 31 August 2006; accepted 8 September 2006

Abstract

A new semi-empirical formula for the evaluation of (n,^{3}He) reaction cross-sections at the energy of 14.6 and 20 MeV is presented. Formula was derived using the analytical expression for the evaluation of the ^{3}He spectrum within the frame of pre-equilibrium exciton model. The systematics obtained is compared with the empirical formula for the (n,^{3}He) reaction cross-section.

Keywords: Activation cross-section; Neutron irradiation; Helium-3; Systematics

The use of systematics for the nuclear reaction cross-section evaluation is crucial if experimental data are scarce and model calculations are unreliable. Systematics are widely used for the evaluation and validation of neutron activation data at the energy around 14 and 20 MeV (Forrest et al., 2005). Empirical and semi-empirical formulae are applied for the creation of systematics. Empirical expressions, as a rule, contain the exponential dependence of cross-sections upon the number of neutrons and protons in nuclei. The formal use of the evaporation model for its justification seems doubtful, because it ignores an important role of the pre-equilibrium mechanism of particle emission for medium and heavy nuclei. The semi-empirical approach to the development of nuclear reaction systematics has been proposed in Bychkov et al. (1978, 1979), Konobeyev and Korovin (1995, 1998, 1999), Konobeyev et al. (1996, 1998), and Dityuk et al. (1996). It is based on the use of analytical expressions for calculation of particle emission spectra within the frame of pre-equilibrium exciton and evaporation models.

In the present work the semi-empirical approach (Konobeyev and Korovin, 1995, 1998, 1999; Bychkov et al., 1978, 1979; Konobeyev et al., 1996, 1998; Dityuk et al., 1996) is used to obtain the systematics of the (n,^{3}He) reaction cross-section at 14.6 and 20 MeV.

Before the systematic study of (n,^{3}He) reaction cross-section has been performed in Qaim (1974, 1978), Qaim et al. (1980), and Lishan and Yuling (1992).

1. Basic principles of the development of the systematics

According to calculations performed (Broeders et al., 2006), the non-equilibrium emission gives the main contribution in the (n,^{3}He) reaction cross-section for nuclei with $A > 40$ at the incident neutron energy below 20 MeV. In this case, the approximate expression for the (n,^{3}He) reaction cross-section can be obtained using the pre-equilibrium exciton model in a "closed form" and the phenomenological pick-up model (Iwamoto and Harada, 1982; Sato et al., 1983). The derivation of the formula is similar to ones performed for the pre-equilibrium component of the (n, α) reaction (Konobeyev et al., 1996),

$$\begin{aligned}\sigma_{(\mathrm{n},^3\mathrm{He})} = \sigma_{\mathrm{non}}(E_\mathrm{n})(4/3)\frac{(2S+1)\mu\sigma_{\mathrm{geom}}}{\pi^3\hbar^2 g^4 E_0^4\langle|M|^2\rangle} \\ \times(E_\mathrm{n}+Q_{(\mathrm{n},^3\mathrm{He})}-V)^3 \\ \times[0.5C_1(E_\mathrm{n}+Q_{(\mathrm{n},^3\mathrm{He})}-V) \\ +C_1(V+Q_{^3\mathrm{He}})+C_2],\end{aligned} \quad (1)$$

*Corresponding author. Tel.: +49 7247 82 2638; fax: +49 7247 82 3718.
E-mail address: konobeev@irs.fzk.de (A.Y. Konobeyev).

0969-8043/$ - see front matter
doi:10.1016/j.apradiso.2006.09.004

where σ_{non} is the cross-section of nonelastic interaction of the incident neutron with a nucleus at the kinetic energy E_n; S, μ and V are spin, reduced mass and Coulomb potential for the outgoing ^{3}He-particle, respectively; $Q_{(n,^3He)}$ is the energy of the (n,^{3}He) reaction; $Q_{^3He}$ is the separation energy for ^{3}He in a compound nucleus; g is the single particle level density; $\langle |M|^2 \rangle$ is the averaged squared matrix element for two-body interaction; E_0 is the energy of the excitation for the compound nucleus; $\sigma_{geom} = \pi R^2$ and R is the nucleus radius; C_1 and C_2 are obtained from the approximation of the ^{3}He form factor from (Sato et al., 1983) by a straight line $F_{1,2} = C_1(\varepsilon + Q_{^3He}) + C_2$, where ε is the kinetic energy of the ^{3}He emitted.

There is a relatively small number of measurements for the (n,^{3}He) reaction cross-section. For this reason, the use of the formula with a few numbers of free parameters is appropriate for the systematics development. Assuming that the averaged squared matrix element for two-body interaction is parameterized as $\langle |M|^2 \rangle = KA^{-3}E_0^{-1}$ (Kalbach, 1978), and the single particle level density g is equal to A/m, where K and m are constants, by analogy with Konobeyev et al. (1996), Dityuk et al. (1996), one can obtain from Eq. (1) the following two parametric formula for the (n,^{3}He) reaction cross-section:

$$\sigma_{(n,^3He)} = \pi r_0^2 (A^{1/3}+1)^2 A^{-1/3} (\alpha_1 (N-Z+1)/A + \alpha_2)^3, \quad (2)$$

where the $(N-Z+1)/A$ dependence results from the use of the semi-empirical mass formula for the energy of the (n,^{3}He) reaction; r_0 is equal to 1.3 fm; N, Z, A are the number of neutrons, protons and nucleons in the target nucleus, respectively; α_i are free parameters.

Considering the uncertainty of the definition of averaged squared matrix element for residual two-body interaction $\langle |M|^2 \rangle$ (Konobeyev and Korovin, 1995), the following three parametric formula can be used for the (n,^{3}He) systematics development

$$\sigma_{(n,^3He)} = \pi r_0^2 (A^{1/3}+1)^2 A^{\alpha_3} (\alpha_1 (N-Z+1)/A + \alpha_2)^3. \quad (3)$$

As mentioned above, both Eqs. (2) and (3) describe the non-equilibrium component of the (n,^{3}He) reaction cross-section.

For the comparison with Eqs. (2) and (3) the systematics from Qaim (1978) is used:

$$\sigma_{(n,^3He)} = \alpha_1 (A^{1/3}+1)^2 \exp(\alpha_2 (N-Z)/A). \quad (4)$$

Eq. (4) can be obtained using the simple evaporation model without taking into account the angular momentum (Lishan and Yuling, 1992). It describes the contribution of the equilibrium emission in the (n,^{3}He) reaction.

The values of free parameters in Eqs. (2)–(4) were obtained from the fitting to experimental cross-sections providing the minimum of the expression

$$\Sigma = \sum_{i=1}^{N} ((\sigma_i^{syst} - \sigma_i^{exp})/\Delta\sigma_i^{exp})^2, \quad (5)$$

where σ_i^{syst} is the cross-section calculated using Eqs. (2)–(4); σ_i^{exp} and $\Delta\sigma_i^{exp}$ are the experimental cross-section and its error; N is the number of target nuclei, for which measurements have been done.

The χ^2 value is calculated as follows:

$$\chi^2 = \Sigma/(N-m), \quad (6)$$

where Σ is calculated through Eq. (5) and m is the number of free parameters.

2. Systematics at 14.6 MeV

Experimental data for the (n,^{3}He) reaction were taken from Qaim (1974, 1978, 1970), Pepelnik et al. (1985), Bahal and Pepelnik (1984), Csikai (1966), Husain et al. (1968), and Csikai and Szalay (1965) and analyzed. Most of the data are available at 14.6 MeV. The cross-sections at other energies (14.7 and 14.8 MeV) were reduced to 14.6 MeV using calculated excitation functions for the (n,^{3}He) reaction (Broeders et al., 2006). Measured cross-sections were renormalized using new data for monitor reactions from Sublet et al. (2003).

The (n,^{3}He) reaction cross-sections at 14.6 MeV obtained from the analysis of experimental data are shown in Table 1. The parameters of Eq. (2)–(4) were fitted to cross-sections obtained. The results are shown in Table 2. The comparison of different formulae shows that Eqs. (2) and (3) have the minimal value of χ^2.

Eqs. (2)–(4) describe contributions of different processes in the (n,^{3}He) reaction cross-section (Section 1). The advantage of systematics proposed in the present work is observed for two- and three-parametric formulae, Eqs. (2) and (3) are compared with Eq. (4) (Table 2). It is an additional argument for the dominant role of the none-quilibrium ^{3}He-emission in the (n,^{3}He) reaction at 14.6 MeV.

Table 1 shows cross-sections calculated using Eq. (3) with parameters from Table 2 and values of $((\sigma_i^{syst} - \sigma_i^{exp})/\Delta\sigma_i^{exp})^2$, which characterizes the deviation of calculated and experimental cross-sections.

3. Systematics at 20 MeV

The experimental data for the (n,^{3}He) reaction around 20 MeV are absent. Cross-sections at this energy were obtained from the model calculations.

The calculation has been performed using the hybrid exciton model combined with the coalescence pick-up model (Sato et al., 1983) and the Weisskopf–Ewing evaporation model. The parameters of models were obtained from the accurate description of the (n,^{3}He)

Table 1
Data used to obtain the (n,^{3}He) reaction cross-section systematics at 14.6 MeV: the neutron energy corresponding to original measured data (third column), original data (fourth column), cross-sections corrected using new data for monitor reactions (Sublet et al., 2003) and reduced to 14.6 MeV using calculated excitation functions (sixth column), adopted cross-sections at 14.6 MeV used for the fitting of free parameters in systematic formulae (seventh column), cross-sections calculated using Eq. (3) with parameters from Table 2 (eight column) and values $((\sigma_i^{syst} - \sigma_i^{exp})/\Delta\sigma_i^{exp})^2$ (last column)

Z	A	Neutron energy (MeV)	Original data (μb)	Reference	Data corrected (μb)	Adopted data $\sigma_i^{exp} \pm \Delta\sigma_i^{exp}$ (μb)	Systematics σ_i^{syst} (μb)	Σ_i
15	31	14.7	30.±10.	Pepelnik et al. (1985)	25.±10.	25.±10.	5.89	3.65
19	41	14.6	6.±3.	Qaim (1978)	5.0±2.5	5.0±2.5	6.34	0.288
21	45	14.6	8.6±4.	Qaim (1974)	8.5±4.	8.5±4.	4.67	0.917
27	59	14.6	4.6±2.1	Qaim (1974)	4.95±2.26	4.95±2.26	3.54	0.387
		14.7	10.4±0.6	Bahal and Pepelnik (1984)	8.91±0.51			
29	63	14.6	4.±2.	Qaim (1978)	3.8±1.9	3.8±1.9	2.85	0.249
33	75	14.6	3.5±1.9	Qaim (1974)	4.15±2.25	4.15±2.25	3.65	0.049
41	93	14.6	3.1±1.5	Qaim (1974)	4.39±2.12	4.39±2.12	2.36	0.915
45	103	14.7	1.2±1.	Csikai (1966)	0.13±0.11	1.44±0.43	2.16	2.84
		14.8	2.±0.6	Husain et al. (1968)	1.44±0.43			
55	133	14.7	5.±3.	Csikai and Szalay (1965)	8.89±5.34	2.30±0.43	2.36	0.023
		14.8	3.2±0.5	Qaim (1970)	2.27±0.35			
58	142	14.6	3.3±1.3	Qaim (1978)	3.56±1.40	3.56±1.40	2.34	0.755
65	159	14.6	4.6±1.8	Qaim (1978)	5.02±1.96	5.02±1.96	1.90	2.54
69	169	14.6	4.±2.	Qaim (1978)	3.90±1.95	3.90±1.95	1.72	1.25
73	181	14.6	3.4±1.5	Qaim (1974)	3.23±1.42	3.23±1.42	1.68	1.18

Table 2
The results of the fitting of Eqs. (2)–(4) to the (n,^{3}He) reaction cross-sections obtained from the analysis of experimental data at 14.6 MeV (Table 1)

Formula	Σ	χ^2	Parameters
(2)	20.8	1.89	$\alpha_1 = 5.8701\times10^{-3}$, $\alpha_2 = 1.7378\times10^{-2}$,
(3)	15.1	1.51	$\alpha_1 = 1.6534$, $\alpha_2 = 0.15257$, $\alpha_3 = -2.3$
(4)	21.8	1.99	$\alpha_1 = 8.475\times10^{-2}$, $\alpha_2 = -1.6467$[a]

[a]Parameters correspond to the cross-section in μb.

Table 3
The (n,^{3}He) reaction cross-section evaluated using available experimental data and results of model calculations (σ_i^{eval}) and cross-sections estimated by the systematics obtained, Eq. (8) (σ_i^{syst}) at the incident neutron energy 20 MeV

Z	A	σ_i^{eval} (μb)	σ_i^{syst} (μb)
15	31	1950	1312
19	41	990	1166
21	45	900	898
27	59	650	639
29	63	540	530
33	75	530	576
41	93	430	379
45	103	270	338
55	133	280	322
58	142	320	311
65	159	330	253
69	169	270	228
73	181	250	219

reaction cross-section at 14.6 MeV (Table 1). Obtained cross-sections at 20 MeV, σ_i^{eval} are shown in Table 3.

The systematics was obtained by the minimization of Σ value, Eq. (5), where σ^{eval} values are used instead of σ^{exp} and σ^{syst} is defined by Eq. (3). The fitting was performed using equal relative errors $\Delta\sigma_i^{eval} = C\sigma_i^{eval}$, where C is the same constant for all target nuclei considered.

After the fitting of Eq. (3) to the data from Table 3, following values of parameters were obtained; $\alpha_1 = 6.4556$, $\alpha_2 = 1.1549$, $\alpha_3 = -2.3$. Table 3 shows the cross-sections estimated using new systematics.

There are two measurements of the (n,^{3}He) reaction cross-section at energies close to 20 MeV (Qaim et al., 1986; Qaim and Wölfle, 1991). Data are available for the ^{93}Nb(n,^{3}He)^{91}Y reaction up to 19.2 MeV (Qaim et al., 1986) and for the ^{139}La(n,^{3}He)^{137}Cs reaction up to 19.0 MeV (Qaim and Wölfle, 1991). The simple comparison with the systematics at 20 MeV cannot be done due to the rapid change of the (n,^{3}He) cross-section with the incident neutron energy at energies below 20 MeV. However, obtained systematics values for ^{93}Nb (379 μb) and ^{139}La (315 μb) are in agreement with general energy trend of cross-sections measured in (Qaim et al., 1986; Qaim and Wölfle, 1991).

4. Results

New systematics have been obtained for the (n,^{3}He) reaction cross-section at the incident neutron energy 14.6 and 20 MeV. The pre-equilibrium exciton model and the phenomenological pick-up model (Sato et al., 1983) were used to get the analytical formula for the cross-section.

The systematics at 14.6 MeV is

$$\sigma_{(n,^3He)} = \pi r_0^2 (A^{1/3}+1)^2 A^{-2.3} (1.6534(N-Z+1)/A + 0.15257)^3 \quad (7)$$

and at 20 MeV

$$\sigma_{(n,^3He)} = \pi r_0^2 (A^{1/3}+1)^2 A^{-2.3} \times (6.4556(N-Z+1)/A + 1.1549)^3, \quad (8)$$

where r_0 is equal to 1.3 fm, which corresponds to the πr_0^2 value equal to 5.3093×10^4 μb; N, Z, A are the number of neutrons, protons and nucleons in the target nucleus, respectively.

The systematics obtained can be used for the evaluation of the (n,^{3}He) reaction cross-section for a wide range of target nuclei.

References

Bahal, B.M., Pepelnik, R., 1984. NEANDC(E)-252/U 28 (5).

Broeders, C.H.M., Konobeyev, A.Yu., Korovin, Yu.A., Lunev, V.P., Blann, M., 2006. ALICE/ASH – pre-compound and evaporation model code system for calculation of excitation functions, energy and angular distributions of emitted particles in nuclear reactions at intermediate energies. FZK 7183: 〈http://bibliothek.fzk.de/zb/berichte/FZKA7183.pdf〉.

Bychkov, V.M., Pashchenko, A.B., Plyaskin, V.I., 1978. Voprosi Atomnoy Nauki i Techniki. Jadernye Konstanty (Nucl. Constants) 4 (31), 48.

Bychkov, V.M., Manokhin, V.N., Pashchenko, A.B., Plyaskin, V.I., 1979. Voprosi Atomnoy Nauki i Techniki. Jadernye Konstanty (Nucl. Constants) 1 (32), 27.

Csikai, J., 1966. Acta Phys. Hung. 21, 229.

Csikai, J., Szalay, A., 1965. Nucl. Phys. 68, 546.

Dityuk, A.I., Konobeyev, A.Yu., Lunev, V.P., Shubin, Yu.N., 1996. Voprosi Atomnoy Nauki i Techniki. Jadernye Konstanty (Nucl. Constants) 1, 129.

Forrest, R.A., Kopecky, J., Sublet, J.-Ch., 2005. UKAEA FUS 515.

Husain, L., Bari, A., Kuroda, P.K., 1968. J. Inorg. Nucl. Chem. 30, 3145.

Iwamoto, A., Harada, K., 1982. Phys. Rev. C 26, 1821.

Kalbach, C., 1978. Z. Phys. A 287, 319.

Konobeyev, A.Yu., Korovin, Yu.A., 1995. Nucl. Instrum. Methods Phys. Res. B 103, 15.

Konobeyev, A.Yu., Korovin, Yu.A., 1998. Atom. Energy 85, 556.

Konobeyev, A.Yu., Korovin, Yu.A., 1999. Nuovo Cimento 112A, 1001.

Konobeyev, A.Yu., Lunev, V.P., Shubin, Yu.N., 1996. Nucl. Instrum Methods Phys. Res. B 108, 233.

Konobeyev, A.Yu., Lunev, V.P., Shubin, Yu.N., 1998. Nuovo Cimento 111A, 445.

Lishan, Y., Yuling, J., 1992. Comm. Nucl. Data Prog. 7, 95.

Pepelnik, R., Anders, B., Bahal, B.M., Farooq, M., 1985. NEANDC(E)-262U 32 (5).

Qaim, S.M., 1970. J. Inorg. Nucl. Chem. 32, 1799.

Qaim, S.M., 1974. J. Inorg. Nucl. Chem. 36, 239 Erratum: ibid., p. 3886.

Qaim, S.M., 1978. Radiochim. Acta 25, 13.

Qaim, S.M., Wölfle, R., 1991. Acta Phys. Hung. 69, 137.

Qaim, S.M., Klapdor, H.V., Reiss, H., 1980. Phys. Rev. C 22, 1371.

Qaim, S.M., Wölfle, R., Liskien, H., 1986. Phys. Rev. C 34, 489.

Sato, K., Iwamoto, A., Harada, K., 1983. Phys. Rev. C 28, 1527.

Sublet, J.-Ch., Koning, A.J., Forrest, R.A., Kopecky, J., 2003. JEFDOC-982.